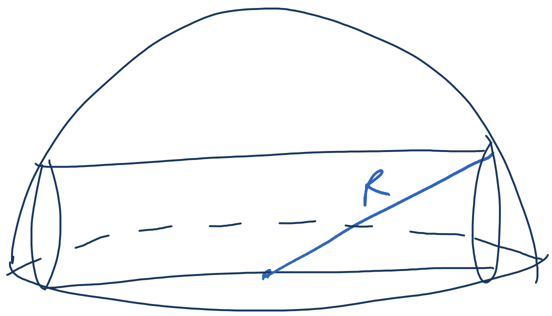
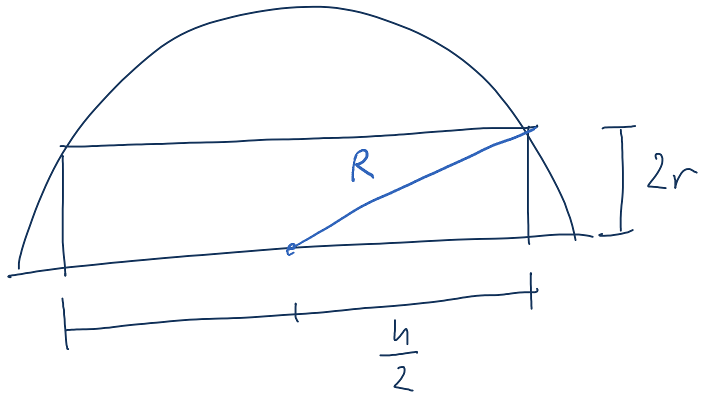
Ma50005:3237

den 21 januari 2014
10:52



Tvärsnitt



Samband: $\left(\frac{h}{2}\right)^2 + (2r)^2 = R^2$

$\Leftrightarrow$

$r^2 = \frac{1}{4}\left(R^2 - \left(\frac{h}{2}\right)^2\right)$

Volym av cylinder (som funktion av h)

$V = V(h) = \pi r^2 h = \pi \cdot \frac{1}{4}\left(R^2 - \left(\frac{h}{2}\right)^2\right) \cdot h =$

$= \frac{\pi}{16}\left(4R^2 h - h^3\right) \quad ; \quad 0 \leq h \leq 2R$

Sök max:

$$V'(h) = \frac{\pi}{4}\left(4R^2 - 3h^2\right) = 0$$

$$\Leftrightarrow$$

$$h = (\pm)\, \frac{2}{\sqrt{3}} R$$

$$V''(h) = \frac{\pi}{4} \cdot \left(-6h\right) \Rightarrow V''\left(\frac{2}{\sqrt{3}} R\right) < 0$$

så $h = \frac{2}{\sqrt{3}} R$ ger lokalt och globalt max.

Största värde $V\left(\frac{2}{\sqrt{3}} R\right) = \frac{\pi}{16}\left(4 \cdot \frac{2}{\sqrt{3}} R^3 - \frac{8}{3\sqrt{3}} R^3\right)$

$$= \frac{\pi}{2\sqrt{3}}\left(R^3 - \frac{1}{3} R^3\right) = \frac{\pi}{2\sqrt{3}} \cdot \frac{2}{3} R^3 = \frac{\pi}{3\sqrt{3}} R^3$$