1104a

den 3 mars 2011
09:10

$$z^2 + (6-2i)z + 5 - 2i = 0$$

Kvadratkomplettera:

$$\underbrace{(z+3-i)^2 - (3-i)^2 + 5 - 2i}_{\text{Fixa till}} = 0$$

$$-(3-i)^2 + 5 - 2i = -9 + 6i + 1 + 5 - 2i = -3 + 4i$$

Så

$$(z+3-i)^2 - 3 + 4i = 0$$

$$\Leftrightarrow$$

$$(z+3-i)^2 = 3 - 4i$$

Sätt $z + 3 - i = w$ så

$$w^2 = 3 - 4i$$

Sätt $w = x + iy$ så

$$(x+iy)^2 = 3 - 4i$$

$$\Leftrightarrow$$

$$x^2 - y^2 = 3$$

$$\begin{cases} x^2 - y^2 = 3 \\ 2xy = -4 \end{cases}$$

$y = -\frac{2}{x}$ ger $x^2 - \frac{4}{x^2} = 3$

$\Leftrightarrow 1$

$$x^4 - 4 = 3x^2$$

Sätt $x^2 = t$ så

$$t^2 - 4 = 3t$$

$\Leftrightarrow 1$

$$t^2 - 3t - 4 = 0$$

$\Leftrightarrow 1$

$$t = \frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 4}$$

$$= \frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}}$$

$$= \frac{3}{2} \pm \frac{5}{2}$$

$t_1 = 4$, $t_2 = -\frac{1}{2}$

förkastas

$t_1 = 4$ ger $x^2 = 4 \Leftrightarrow x = \pm 2$

$t_1 = 4$ ger $x^2 = 4 \Leftrightarrow x = \pm 2$

$x = 2$ ger $y = -\frac{2}{2} = -1$

$x = -2$ ger $y = -\frac{2}{-2} = 1$

Så $w_1 = 2 - i$, $w_2 = -2 + i$

$w = z + 3 - i \Leftrightarrow z = w - 3 + i$ så

$z_1 = -1$, $z_2 = -5 + 2i$