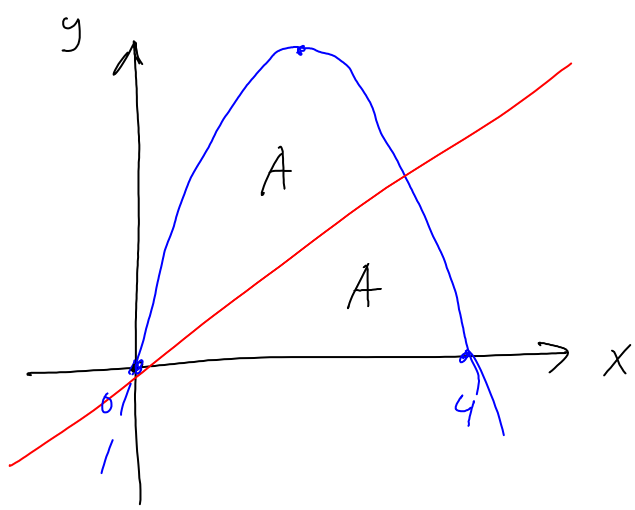
2320

den 12 februari 2010
10:38

Skiss av graf till $y = 4x - x^2$.



Vi väljer en linje genom origo:

$y = kx$. (enklast?)

När skär linjen kurvan? Jo då

$$4x - x^2 = kx \iff x^2 + (k-4)x = 0$$

$$\iff x(x+k-4) = 0$$

så antingen $x = 0$ eller $x = 4-k$

"no surprise"

Arean under $y = 4x - x^2$, $0 \le x \le 4$:

$$2A = \int_0^4 (4x - x^2)\,dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4 =$$

$$2A = \int_0^4 (4x - x^2)\,dx = \left[2x^2 - \frac{x^3}{3} \right]_0^4 =$$

$$32 - \frac{64}{3} = \frac{32}{3}$$

så

$$A = \frac{16}{3}$$

Alltså

$$\int_0^{4-k} (4x - x^2 - kx)\,dx = \frac{16}{3}$$

$$= \left[2x^2 - \frac{x^3}{3} - \frac{kx^2}{2} \right]_0^{4-k} =$$

$$= 2(4-k)^2 - \frac{(4-k)^3}{3} - \frac{k(4-k)^2}{2} =$$

$$= (4-k)^2 \left(2 - \frac{4-k}{3} - \frac{k}{2} \right) =$$

$$= (4-k)^2 \left(\frac{2}{3} - \frac{k}{6} \right) = (4-k)^2 \cdot \frac{1}{6}(4-k) =$$

$$= \underbrace{\frac{1}{6}(4-k)^3}_{\text{vilken "tur"!}} = \frac{16}{3}$$

$$\Leftrightarrow$$

$$(4-k)^3 = 32$$

$$(4-k)^3 = 32$$

$$\Leftrightarrow$$

$$4-k = 32^{1/3}$$

$$\Leftrightarrow$$

$$k = 4 - 32^{1/3}$$