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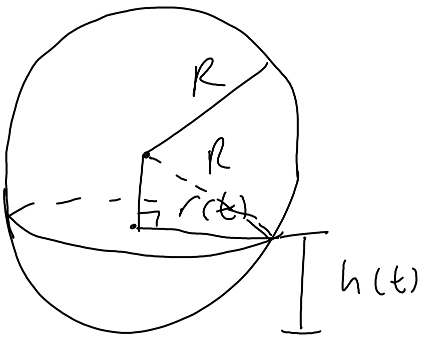
19:59

Lsg 1



Observera att

$$R^2 = r(t)^2 + (R - h(t))^2$$

$$\iff$$

$$r(t)^2 = 2R \cdot h(t) - h(t)^2$$

Känt: $V'(t) = 25 \ (l/min) = 0{,}025 \ (m^3/min)$

$h(t_0) = 0{,}5 \ (m)$

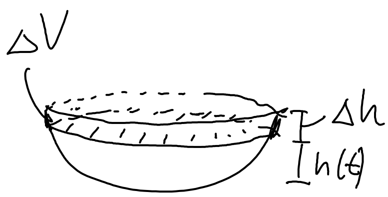
a) Låt $h(t + \Delta t) = \Delta h$

$V(t + \Delta t) = \Delta V$

Vi söker $\lim\limits_{\Delta t \to 0} \dfrac{\Delta h}{\Delta t} = \dfrac{dh}{dt} = h'(t)$

då $t = t_0$.

Principfigur



Om ΔV litet ("tunn skiva") fås

$$\Delta V \approx \pi r(t)^2 \cdot \Delta h$$

$$\iff$$

$$\Delta V \approx \pi \cdot \left(2Rh(t) - h(t)^2\right) \cdot \Delta h$$ enl. tidigare samband

$$\Leftrightarrow$$

$$\frac{\Delta V}{\Delta t} \approx \pi \cdot \left(2Rh(t) - h(t)^2\right) \cdot \frac{\Delta h}{\Delta t}$$ dela med Δt

$t = t_0$ ger $h(t) = h(t_0) = 0{,}5$, $R = 5$
och $\Delta V / \Delta t = 0{,}025$. Insättning:

$$0{,}025 \approx \pi \cdot \left(2 \cdot 5 \cdot 0{,}5 - 0{,}5^2\right) \cdot \frac{\Delta h}{\Delta t}$$

Om $\Delta t \to 0$ blir approximationen godtyckligt bra ($\approx$ blir $=$) och vi får

$$0{,}025 = \pi \left(5 - 0{,}25\right) \cdot h'(t_0)$$

$$\Leftrightarrow$$

$$h'(t_0) = \frac{0{,}025}{4{,}75\,\pi}$$

b)

$$V(t) = \frac{1}{3} \pi h(t)^2 \left(3R - h(t)\right)$$

$$\Rightarrow$$

$$V'(t) = \frac{1}{3} \pi \Big(2h(t) \cdot h'(t) \cdot \left(3R - h(t)\right) +$$

$$+ h(t)^2 \cdot \left(-h'(t)\right)\Big)$$

(produkt- och kedjeregel)

$V'(t) = 0{,}025$, $h(t_0) = 0{,}5$, $R = 5$ ger

$$0{,}025 = \frac{1}{3} \pi \cdot 2 \cdot 0{,}5 \cdot h'(t_0) \cdot \left(15 - 0{,}5\right) +$$

$$\left(+ 0{,}25 \cdot (-h'(t_0)) \right)$$

$$\Leftrightarrow$$

$$0{,}025 = \frac{1}{3}\pi \cdot h'(t_0) \cdot \left(14{,}5 - 0{,}25 \right)$$

$$\Leftrightarrow$$

$$h'(t_0) = \frac{0{,}075}{\pi} \cdot \frac{1}{14{,}25} = \frac{0{,}025}{4{,}75\pi}$$